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LETTER TO THE EDITOR

Retrieval via non-equilibrium states in neural networks

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Abstract. We describe a simple variant of Hopfield's or Little's model for neural networks, which is able to act as an associative memory even in the heavily overloaded spin-glass phase, using non-equilibrium states as attractors.

The storage capacity and retrieval properties of fully connected neural networks have been intensively studied both by numerical simulations and by equilibrium statistical mechanics (Amit *et al* 1987).

One of the outstanding limitations of thermodynamical calculations is the impossibility of describing states with finite correlations with all memorised patterns. The number of these states, called non-equilibrium states, since they are invisible to thermodynamics, grows exponentially with the number of neurons N (Gardner 1986). They end up invading the basin of attraction of the retrieval states, i.e. states having finite overlap with only one memorised pattern (prototype).

In this letter we address the question of whether there exist, or how to produce, non-equilibrium retrieval states for the Hopfield (1982) or Little (1974) models. Obviously these states have to be different from the ones mentioned above, since they have finite correlations with only one of the prototypes. Both though are invisible to thermodynamics.

For simplicity we restrict ourselves to the zero temperature, deterministic case. If the neuron's states are represented by Ising spins $S_i = +1$ for active and $S_i = -1$ for passive, the dynamics is governed by the equations

$$S_i(t+1) = \operatorname{sgn}(h_i(t)) \qquad i = 1, \ldots, N \tag{1a}$$

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j(t) \tag{1b}$$

with

$$J_{ij} = \frac{1}{N} \sum_{\mu=1}^{P} \xi_i^\mu \xi_j^\mu \qquad i \neq j \tag{1c}$$

where $\{\xi_i^\mu = \pm 1, i = 1, \ldots, N\}$ is a set of P uncorrelated prototypes.

This model has two phases characterised by the retrieval overlap

$$m = \frac{1}{N} \left\langle \sum_i \xi_i^\mu S_i(t=\infty) \right\rangle_\xi \tag{2}$$

where $\langle\ \rangle_\xi$ stands for an average over initial configurations and over sets of prototypes. In the retrieval phase we get $m \geqslant 0.97$, whereas in the spin-glass phase $m \leqslant 0.45$. The discontinuous transition between these phases occurs at $\alpha = \alpha_c = 0.145 \pm 0.01$, where $\alpha = P/N$ (Amit *et al* 1987).

This letter is motivated by the observation of the following interesting fact, which happens in the spin-glass phase. If the system is initialised in the vicinity of a prototype, then it starts evolving in the direction of this prototype, but eventually gets captured in one of the spin-glass attractors (Gardner *et al* 1987). A similar behaviour is found in the retrieval phase for very noisy input patterns (van Hemmen 1987). Thus there exists a latent retrieval capacity which could be tapped. The aim would primarily be not so much the increase in storage capacity, although this does happen, but the possibility of retrieving very noisy patterns (van Hemmen 1987). In order to achieve this goal, we include a kind of friction term in our model, so that the system gets trapped in one of the many metastable states surrounding every prototype (Gardner 1986, Treves and Amit 1988). This can be done with extreme simplicity, turning on a diagonal coupling

$$J_{ii} = J_0 > 0. \tag{3}$$

Obviously J_0 does not change the thermodynamics of the Hopfield model, whose Hamiltonian acquires an additive constant. Although less obvious, this also holds for Little's model in the $T \to 0$ limit (Fontanari and Köberle 1987, 1988). The evolution equation (1*b*) becomes

$$h_i(t) = \sum_{j \neq i} J_{ij} S_j(t) + J_0 S_i(t). \tag{4}$$

The term $J_0 S_i(t)$ increases the stability of any configuration for $J_0 > 0$.

The effect of a positive self-interaction has been discussed in the context of a non-local model due to Personnaz *et al* (1986) (see also Kanter and Sompolinsky 1987): the basin of attraction of the retrieval states is severely reduced. However this assertion assumes implicitly the existence of these states.

What we will show by numerical simulations is that for appropriately chosen values of J_0 we can create retrieval states, where they did not exist before, i.e. in the spin-glass phase. Increasing J_0 beyond this value introduces damaging effects as also observed in the model of Personnaz *et al.*

In figure 1 we show the equilibrium retrieval overlap (2) as a function of the initial retrieval overlap

$$m_0 = \frac{1}{N} \left\langle \sum_i \xi_i^\mu S_i(t=0) \right\rangle_\xi \tag{5}$$

for $\alpha = 0.1$, 0.3 and 0.4. Maintaining the realisation of prototypes and initial configurations we repeat the experiments for several values of J_0.

For $\alpha = 0.1$, figure 1(*a*), the system remembers if $m_0 \geqslant 0.4$, since m is nearly equal to 1. On the other hand, as expected, for large values of J_0, e.g., $J_0 \geqslant 1$ the system does not evolve much, becoming useless as an associative memory. The best retrieval capacity was found for $J_0 \approx 0.25$.

For appropriate values of J_0, the system remembers even for $\alpha = 0.3$ or $\alpha = 0.4$ as shown in figures 1(*b*) and 1(*c*).

In figure 2 we show that there exists, for fixed α, a best value for J_0 which maximises the equilibrium retrieval correlation.

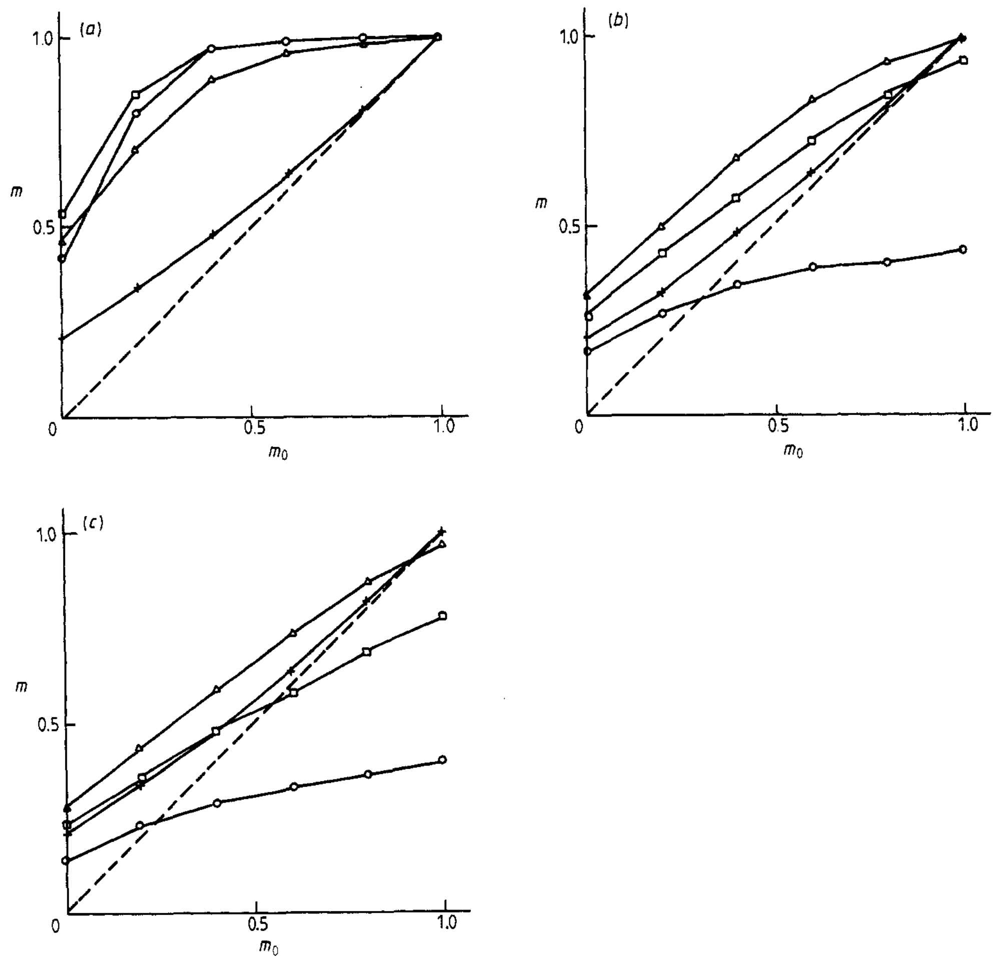

Figure 1. The equilibrium retrieval overlap m as a function of the initial overlap m_0, for $J_0 = 0$ (○), 0.25 (□), 0.5 (△) and 1.0 (+), for $N = 200$. (a) For $\alpha = 0.1$ the system remembers except for $J_0 = 1$. (b) For $\alpha = 0.3$ the system still remembers for $J_0 = 0.25$ and 0.5 although the retrieval is poorer than for $\alpha = 0.1$. (c) For $\alpha = 0.4$ the system remembers only for $J_0 = 0.5$. The broken lines are $m = m_0$. Each point is averaged over 1000 initial states.

In order to display the ability of J_0 to trap the system in a non-equilibrium state for small evolution time, we show in figure 3 the relaxation time plotted against J_0. For $J_0 > 0$ and $\alpha > 0.145$ equilibrium is attained much faster than for $J_0 = 0$. We believe that the increase in relaxation time for $\alpha < 0.145$ is due to a competition between thermodynamic and non-equilibrium retrieval states.

Simulations for asynchronous models yield essentially the same results.

Although the increase in storage capacity is much smaller than that obtained by modifying the learning rule (Gardner 1987), our method is an extremely simple way to put latent retrieval states to good use in the Hopfield-Little model and allows retrieval of patterns in the presence of a large amount of noise.

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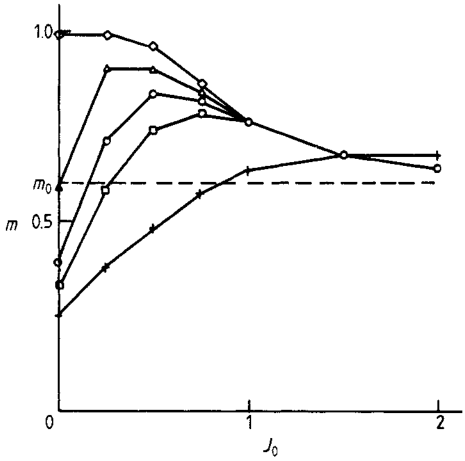

Figure 2. The equilibrium retrieval overlap m plotted against for $\alpha = 0.1$ (◇), 0.2 (△), 0.3 (○), 0.4 (□) and 1.0 (+). The initial state has an overlap $m_0 = 0.6$ with one of the prototypes, and $N = 200$.

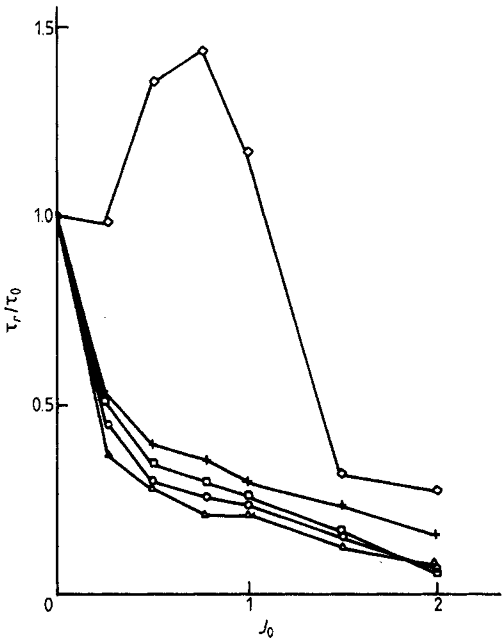

Figure 3. The normalised relaxation time τ_r/τ_0 plotted against J_0 for $\alpha = 0.1$ (◇), 0.2 (△), 0.3 (○), 0.4 (□) and 1.0 (+), where $\tau_0 = \tau_0(\alpha) \equiv \tau_r(J_0 = 0, \alpha)$. The initial overlap is $m_0 = 0.6$, and $N = 200$.

References

Amit D J, Gutfreund H and Sompolinsky H 1987 *Ann. Phys., NY* **173** 30
Fontanari J F and Köberle R 1987 *Phys. Rev.* A **36** 2475
—— 1988 *J. Physique* **49** 13
Gardner E 1986 *J. Phys. A: Math. Gen.* **19** L1047
—— 1987 *Europhys. Lett.* **4** 863
Gardner E, Derrida B and Mottishaw P 1987 *J. Physique* **48** 741
Hopfield J J 1982 *Proc. Natl Acad. Sci., USA* **79** 2554
Kanter I and Sompolinsky H 1987 *Phys. Rev.* A **35** 380
Little W A 1974 *Math. Biosci.* **19** 101
Personnaz L, Guyon I and Dreyfus G 1986 *Phys. Rev.* A **34** 4217
Treves A and Amit D J 1988 *Preprint* Institute for Advanced Studies, Jerusalem, 88/119
van Hemmen J L 1987 *The Physics of Structure Formation* (*Lectures Notes in Physics* **275**) ed W Güttinger and G Dangelmayr (Berlin: Springer) p 547